

Cross Strand Investigation

Building Castles



You will need Pattern Blocks.
Be sure you have squares and triangles.



Part 1

Look at this pattern.



Frame 1



Frame 2



Frame 3

How many squares are in each frame?
How many triangles are in each frame?

Each block has a side length of 1 unit.
What is the perimeter of each frame?

Record the frame number, number of squares, number of triangles,
and perimeter in a table.

Part 2

- Build Frame 4.
How many squares and triangles did you use?
What is the perimeter?
Record the data in your table.
- How many squares and triangles will you need to build Frame 5?
How did you find out?
Build Frame 5 to check your prediction.
- Predict the number of squares and triangles needed to build Frame 10.
How did you make your prediction?
- Write each pattern rule:
 - the numbers of squares in the frames
 - the numbers of triangles in the frames
 - the perimeters of the frames



Display Your Work

Record your work.
Describe the patterns you discovered.

Take It Further

Choose three different Pattern Blocks.
Build your own pattern.
Sketch the first 4 frames.
What number patterns can you find?

Number Patterns

1

Charity Fundraising

Learning Goals

- find a pattern rule for a number pattern
- identify, extend, and create patterns
- describe and model patterns
- use patterns to pose and solve problems
- use patterns in a table to make predictions

Key Words

Input/Output machine

operation

Every September, people around the world take part in the Terry Fox Run. The run raises millions of dollars for cancer research.

This September, Carl will run 10 km.

Carl made this table to find out how much he would get from each pledge.

Amount per Kilometre	Amount of Pledge
10¢	\$1.00
20¢	\$2.00
30¢	\$3.00
40¢	\$4.00
50¢	\$5.00

Carl will run around a 400-m track.

Here is part of a table. It shows how many laps Carl needs to complete, to run 10 km.

Number of Laps	Distance in Kilometres
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0

- What patterns do you see in the tables?
- One of Carl's friends pledged 60¢ per kilometre. What is the amount of this pledge?
- How could you find out how many laps Carl will run?

1

Number Patterns and Pattern Rules

Is each pattern a growing pattern, a shrinking pattern, or a repeating pattern?

- 9, 14, 19, 24, 29, ...
- 8, 5, 1, 8, 5, 1, ...
- 48, 46, 44, 42, 40, ...
- 4, 8, 16, 32, 64, ...

What is each pattern rule?

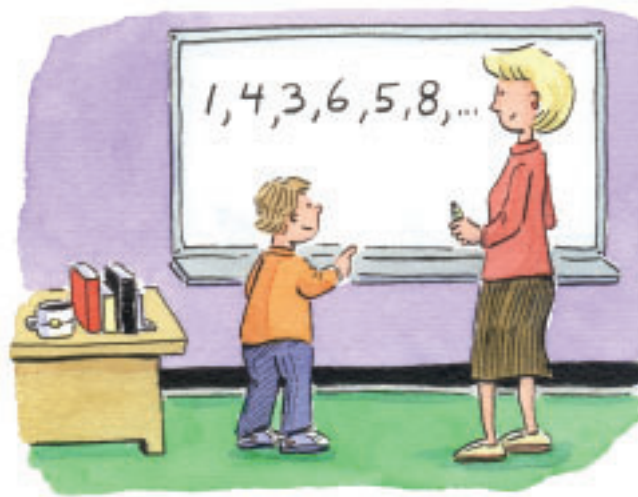
Explore



For each number pattern:

- 3, 4, 6, 9, 13, ...
- 3, 4, 6, 7, 9, ...
- 1, 4, 3, 6, 5, 8, ...
- 1, 2, 5, 10, 17, 26, ...

- Identify the pattern rule.
Write the next 5 terms.
- Make up a similar pattern.
Trade patterns with a classmate.
Write the rule for each of your classmate's patterns.



Show and Share

Share your patterns with other classmates.
How do you know each pattern rule is correct?
For any pattern, did you find more than one rule?
Explain.

Connect

- Here is a number pattern.

$$\begin{array}{ccccccccc} 5 & 6 & 8 & 11 & 15 & \dots \\ \curvearrowright & \curvearrowright & \curvearrowright & \curvearrowright & & & & & \\ +1 & +2 & +3 & +4 & & & & & \end{array}$$

The pattern rule is:

Start at 5. Add 1.

Increase the number you add by 1 each time.

To get the next 5 terms, continue to increase the number you add by 1 each time.

5, 6, 8, 11, 15, 20, 26, 33, 41, 50, ...

- Here is another number pattern.

$$\begin{array}{ccccccccc} 2 & 5 & 9 & 12 & 16 & \dots \\ \curvearrowright & \curvearrowright & \curvearrowright & \curvearrowright & & & & & \\ +3 & +4 & +3 & +4 & & & & & \end{array}$$

The pattern rule is:

Start at 2. Alternately add 3, then add 4.

To get the next 5 terms, continue to add 3, then add 4.

2, 5, 9, 12, 16, 19, 23, 26, 30, 33, ...

- Here is another number pattern.

$$\begin{array}{ccccccccc} 10 & 6 & 11 & 7 & 12 & \dots \\ \curvearrowright & \curvearrowright & \curvearrowright & \curvearrowright & & & & & \\ -4 & +5 & -4 & +5 & & & & & \end{array}$$

The pattern rule is:

Start at 10. Alternately subtract 4, then add 5.

To get the next 5 terms, continue to subtract 4, then add 5.

10, 6, 11, 7, 12, 8, 13, 9, 14, 10, ...

To identify the pattern rule, I find the difference between pairs of consecutive numbers in the pattern.



Numbers Every Day

Number Strategies

Estimate each difference.

$$357 - 85$$


$$423 - 176$$

$$5652 - 609$$

Which strategies did you use?

Practice

Use a calculator when it helps.

- Write the first 5 terms of each pattern.
 - Start at 3. Add 9 each time.
 - Start at 5. Add 2. Increase the number you add by 2 each time.
 - Start at 7. Alternately add 3, then subtract 1.
- Write the next 4 terms in each pattern. Write each pattern rule.
 - 1, 2, 4, 5, 7, 8, ...
 - 2, 4, 3, 5, 4, 6, 5, ...
 - 98, 85, 87, 74, 76, ...
 - 1, 10, 7, 70, 67, 670, ...
- Find each missing term. Write the pattern rule.
 - 3, 23, 13, 33, \square , 43, 33, ...
 - 99, 98, 198, 197, \square , 296, 396, ...
 - 2, 22, 12, 132, 122, 1342, \square , ...
- What is the 7th term of this pattern?
Start at 200. Subtract 8 each time.
How could you find the 7th term without writing the first 6 terms?
- Find each missing term. Write the pattern rule.
 - 74, 148, 222, \square , 370, ...
 - 100, 198, 295, 391, \square , 580, ...
 - 1122, 1112, 1101, 1091, 1080, \square , 1059, ...
- What is the 10th term of this pattern?
Start at 13. Alternately subtract 4, then add 5.
-  The first 2 terms of a pattern are 6, 12,
How many different patterns can you write with these 2 terms?
For each pattern, list the first 6 terms and write the pattern rule.
Show your work.

Reflect

How do you find the pattern rule for a number pattern?
Use an example to explain.

2

Creating Number Patterns

Look at this **Input/Output machine**.

Any number that is put into this machine is multiplied by 5.

If you input 5, the output is 25.

If you input 9, the output is 45.



Explore



- Draw your own Input/Output machine.
Choose a number to go inside your machine.
Choose an operation.
Use your machine to create a number pattern.
- Copy and complete this table for your pattern.
Write the pattern rule for the output numbers.

An **operation** is add, subtract, multiply, or divide.

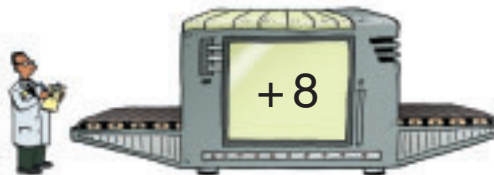
Input	Output
1	
2	
3	

Show and Share

Share your machine and table with another pair of classmates.

Use your classmates' machine to extend their number pattern.

Connect



We can use an Input/Output machine to make a growing pattern.

Each input is added to 8 to get the output.

When we input 1, the output is 9.

When we input 2, the output is 10.

The pattern rule for the output is:

Start at 9. Add 1 each time.

Input	Output
1	9
2	10
3	11
4	12

Practice

Use a calculator when it helps.

1. For each Input/Output machine:

- Choose 5 numbers to input.
- Find each output number.
- Copy and complete the table.

Input	Output

a) Input →  → Output


b) Input →  → Output

2. Copy and complete the table for this Input/Output machine.

Input →  → Output

Input	Output
20	
19	
18	
17	
16	

3. a) Copy and complete the table for this Input/Output machine.

Input →  → Output

Input	Output
36	
42	
48	
54	
60	
66	
72	

b) What is the pattern rule for the input numbers? The output numbers?

Math Link

Number Sense

When an Input/Output machine uses multiplication, and the input numbers are whole numbers, the output numbers are multiples of the number in the machine.

4. Each table shows the Input/Output from a machine.
- Identify the number and the operation in the machine.
 - Continue the patterns.

Write the next 4 input and output numbers for each table.

a)

Input	Output
2	20
4	40
6	60
8	80
10	100

b)

Input	Output
500	485
450	435
400	385
350	335
300	285

5. For each table:

- Write the pattern rules for the output numbers and the input numbers.
- The patterns continue. Write the next 4 input and output numbers.

a)

Input	Output
110	101
99	90
88	79
77	68
66	57

b)

Input	Output
84	12
91	13
98	14
105	15
112	16
119	17



6. Draw an Input/Output machine. Choose a number and an operation. Use multiples of any number as the input numbers. Find the output numbers. Make a table to show your results. Write the pattern rules for the input numbers and the output numbers.

Reflect

When you see an Input/Output table, how can you tell what the operation is in the machine? Use words, pictures, or numbers to explain.

Numbers Every Day

Mental Math

Add.

$$156 + 302$$

$$411 + 298$$

$$2356 + 1009$$

Which strategies did you use?

3

Modelling Patterns

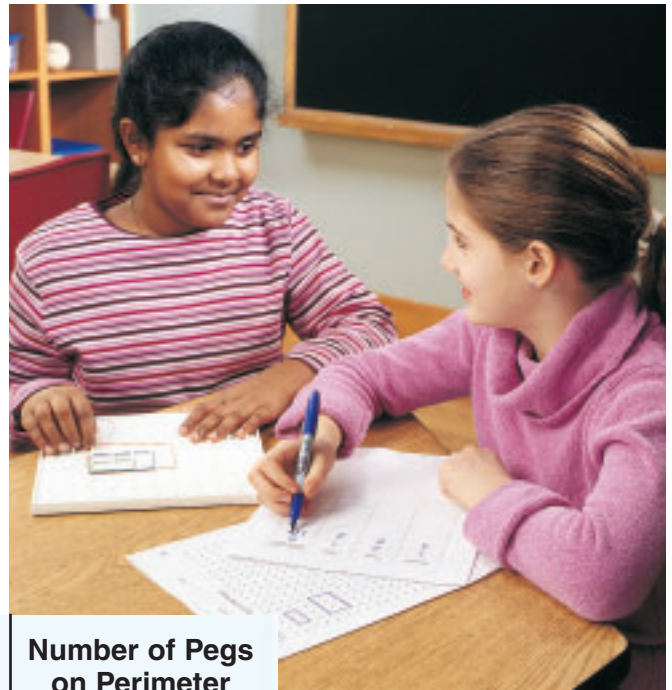
Explore



You will need a geoboard, geobands, and dot paper.

- Use the geoboard to make a rectangle with length 2 units and width 1 unit. Count and record the number of pegs on the perimeter of the rectangle.
- Make a rectangle with length 3 units and width 2 units. Count and record the number of pegs on the perimeter.
- Continue to make rectangles with length 1 unit greater than the width. Record the length, the width, and the number of pegs each time.

Draw each rectangle on dot paper.



Rectangle	Length	Width	Number of Pegs on Perimeter
1	2	1	6
2	3	2	

- How many pegs will be on the perimeter of the 10th rectangle? The 20th rectangle?
- Will the perimeter of any rectangle have 42 pegs? 44 pegs? 46 pegs? How do you know?

Show and Share

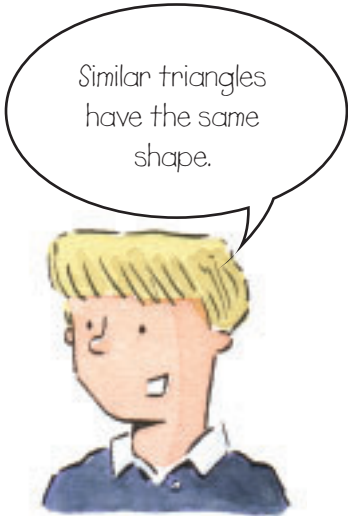
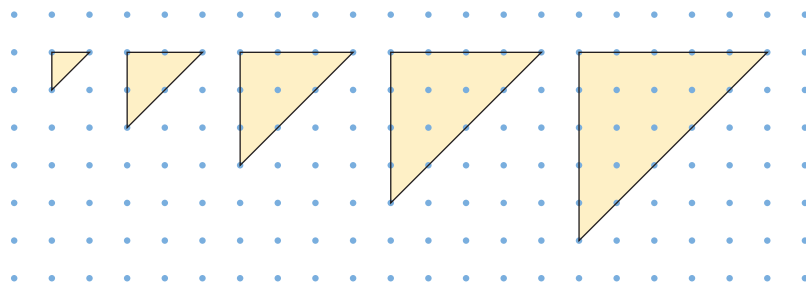
Share your results with another pair of classmates.

What patterns do you see in the table?

How did you use these patterns to solve the problems?

Connect

Here is a pattern of similar triangles drawn on dot paper. Each triangle has 2 equal sides.



This pattern continues.

- Find the number of dots on the perimeter of the 10th triangle and the 20th triangle.

Make a table.

Triangle	Number of Dots on Perimeter
1	3
2	6
3	9
4	12
5	15

These numbers are multiples of 3.

One pattern rule for the number of dots on the perimeter is:

Start at 3. Add 3 each time.

Another pattern rule for the number of dots is:

The triangle number multiplied by 3

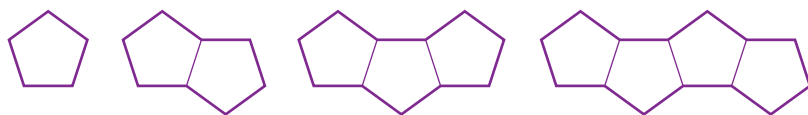
The 10th triangle will have 10×3 , or 30 dots on its perimeter.
 The 20th triangle will have 20×3 , or 60 dots on its perimeter.

- Will any triangle have a perimeter of 50 dots? 51 dots? Use the pattern to find out.

The number of dots on any perimeter is a multiple of 3. 50 is not a multiple of 3, so no triangle has 50 dots. 51 is a multiple of 3 because $17 \times 3 = 51$. So, the 17th triangle has 51 dots.

Practice

1. Regular pentagons are combined to make new figures.
Each pentagon touches no more than 2 other pentagons.



The side length of each pentagon is 1 unit.
The perimeter of each figure is recorded in a table.

Number of Pentagons	Perimeter (units)
1	5

- Copy and complete the table for the first 4 figures.
 - Write the pattern rule for the perimeters.
 - Use the pattern to predict the perimeter of the figure with 10 pentagons. With 20 pentagons.
2. Here is a pattern of figures made with Colour Tiles.

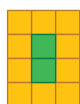


Figure 1

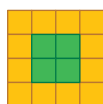


Figure 2

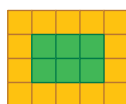


Figure 3

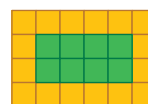


Figure 4

The pattern continues.

- Draw the next two figures on grid paper.
- Copy and complete the table for the first 6 figures.

Figure	Number of Green Tiles	Number of Yellow Tiles
1	2	10

- Write the pattern rule for the number of green tiles.
The number of yellow tiles.
- How many green tiles will be in the 15th figure?
- How many yellow tiles will be in the 20th figure?
- Will any figure have 31 green tiles? 41 yellow tiles? Explain.

3. Pizza Parlour has prices that follow a pattern.

Pizza Number	Pizza Size	Price with Cheese (\$)	Price with Two Toppings (\$)
1	4-slice	5	7
2	8-slice	10	12
3	12-slice	15	17
4	16-slice	20	
5	20-slice		

- Copy and complete the table.
- Write a pattern rule for the price with cheese.
- Write a pattern rule for the price with two toppings.
- The patterns continue.
What is the price of a 24-slice pizza with two toppings?
- How is the price with two toppings related to the price with cheese?



4. Dreamy Ice-Cream Company sells a single-scoop sundae for \$1.25. Each additional scoop costs \$1.25. There is a fixed price of 50¢ for extra topping on all sundaes.
- Make a table to show the prices of the first 4 sizes of sundaes with extra topping.
 - Write a pattern rule for any patterns you see.
 - What is the price of a 6-scoop sundae with extra topping? Explain. Show your work.



Reflect

Use a question from this lesson.
How can patterns in a table help you solve a problem, without extending the table?
Use words, pictures, or numbers to explain.

Numbers Every Day

Number Strategies

Order the decimals in each set from least to greatest.

2.3, 1.3, 10.1

1.98, 2.01, 1.89

0.35, 0.23, 2.30

4

Using Patterns to Solve Problems

Explore

Sam baby-sits to make money.
Sam charges \$6 for each hour he works.

- How much does Sam earn when he works 2 hours? 3 hours? 4 hours? 5 hours?
Show your results in a table.

Time Worked (hours)	Money Earned (\$)



- What patterns do you see in the table?
Write rules for these patterns.
Use the patterns to predict how much Sam will earn working 21 hours.
- Sam saves all the money he earns.
He needs \$250 to buy a mountain bike.
How many hours does Sam need to work?
- Make up your own problem you can solve using this table.
Trade problems with a classmate.
Solve your classmate's problem.

Show and Share

Share your answers with a classmate.
Did you solve the problems the same way? Explain.

Connect

One puzzle book costs \$17.

- How much does it cost to buy 2 books? 3 books? 4 books?

Make a table.

When you add 1 to the number of books, you add \$17 to the cost.

Two books cost \$34.
Three books cost \$51.
Four books cost \$68.

Number of Books	Cost (\$)
1	17
2	34
3	51
4	68

These numbers are multiples of 17.

- Use a pattern to predict the cost of 20 books.

One pattern rule for the cost is:

Start at 17. Add 17 each time.

Another pattern rule for the cost is:

The number of books multiplied by 17.

To predict the cost of 20 books, multiply: $20 \times 17 = 340$
Twenty books cost \$340.

- Extend the pattern to find how many books you can buy with \$200.
Use a calculator.

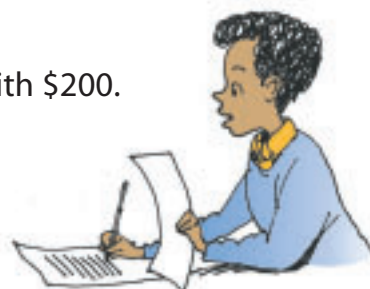


Press: ...

Record each number in the table.

Continue to press until the number is greater than 200.

Eleven books cost \$187.
Twelve books cost \$204.
So, you can buy 11 books with \$200.



Number of Books	Cost (\$)
1	17
2	34
3	51
4	68
5	85
6	102
7	119
8	136
9	153
10	170
11	187
12	204

Practice

1. The pattern in this table continues.

Number	Cost (\$)
1	16
2	32
3	48
4	64
5	
6	
7	

- a) Copy and complete the table.
 b) Write a pattern rule for the cost.
 c) Write a story problem you could solve using this table.
 Solve your problem.
2. Jen has a paper route. Each week she collects \$31.
 a) How much money has Jen collected at the end of 1 week? 2 weeks?
 b) Make a table to show the amounts for the first 8 weeks.
 c) How much will Jen collect in total in 3 weeks?
 d) How much will Jen collect in 1 year?
 e) Write a problem you could solve using the table in part a.
 Solve your problem.
3. The sunflower is the only single flower that grows as high as 3 m.
 It can grow 30 cm each week.
 In which week could a sunflower reach a height of 3 m? Explain.

Numbers Every Day

Mental Math

Multiply.

$$5 \times 30$$

$$60 \times 4$$

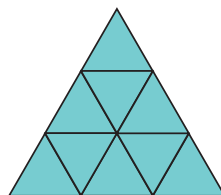
$$700 \times 6$$

$$9 \times 8000$$

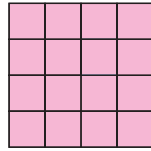
Which strategies did you use?



4. Look at this figure.
 a) How many triangles are there with side length 1 unit? 2 units? 3 units?
 b) How many triangles are in this figure?



5. How many squares are in this figure?



6. Dave read 70 pages on Monday, 67 pages on Tuesday, and 64 pages on Wednesday.

This pattern continued until Dave finished his book.

- a) Make a table to show how many pages were read each day.
- b) What was the total number of pages Dave read the first 7 days?
- c) Dave finished his book on the day he read 31 pages.

How many pages are in the book?

Show your work.

7. Greenholme Elementary School has a car wash to raise money for the community.

The students charge \$4 to wash a car.

- a) Make a table to show how much money is collected when these numbers of cars are washed:

10, 20, 30, 40, 50

The students spent \$23 for soap and sponges.

This amount is subtracted from the amount collected.

- b) How many cars have to be washed to raise \$350?
- c) Write your own problem about this car wash.

Solve the problem you posed.



Reflect

How can using patterns help you solve problems? Use an example from this lesson to explain.

At Home



What number patterns do you see at home? Look through magazines and newspapers. Write about the patterns you see.

5

Strategies Toolkit

Explore



Abi made an Input/Output machine that uses two operations.

Here is a table for Abi's machine.

Find out what the machine does to each input number.



Input	Output
1	5
2	7
3	9
4	11
5	13

Show and Share

Explain the strategy you used to solve the problem.

Connect

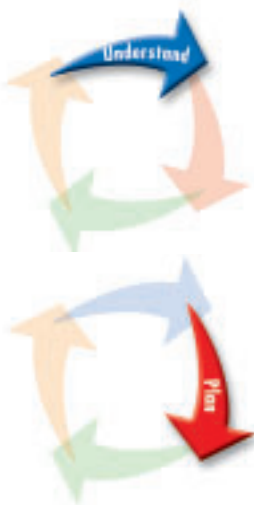
Ben made an Input/Output machine that uses two operations.

Here is a table for Ben's machine. What does Ben's machine do to each input number?

Input	Output
1	2
2	5
3	8
4	11
5	14

Strategies

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.

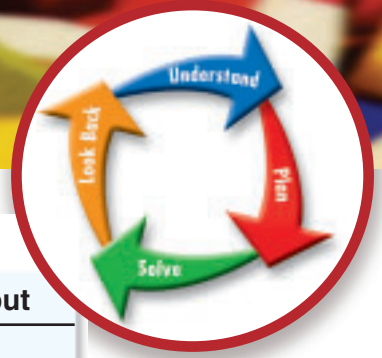


What do you know?

- The machine uses two operations on an input number. The operations could be add, subtract, multiply, or divide.

Think of a strategy to help you solve the problem.

- You can **use a pattern**.
- Analyse the pattern in the *Output* column to find out what the machine does to each input number.



The numbers in the *Output* column increase by 3.
 This suggests the pattern involves multiples of 3.
 Which two operations does Ben's machine use?

Input	Output
1	2
2	5
3	8
4	11
5	14



Find the pattern rule for the output numbers.
 Extend this pattern.
 Use the operations in the machine to extend the pattern of the output numbers.
 Check the numbers in both patterns match.

Practice

Choose one of the Strategies

- Design an Input/Output machine that gives these results.
 How did you decide which operations to use?
- Jolene and her 6 friends exchange friendship bracelets.
 How many bracelets are needed so each person gets a bracelet from each other person?

Input	Output
1	9
2	14
3	19
4	24
5	29

Reflect

Choose one of the questions in this lesson.
 Explain how you used a pattern to solve it.

LESSON

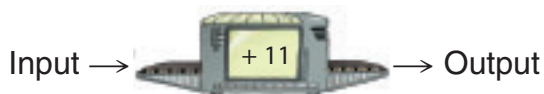
1. Write the first 6 terms of each pattern.
 - a) Start at 100. Subtract 9 each time.
 - b) Start at 10. Alternately add 5, then multiply by 2.

2. Write the next 5 terms in each pattern.

Write the pattern rule.

- a) 5, 8, 12, 15, 19, ...
- b) 50, 48, 47, 45, 44, ...
- c) 10, 12, 16, 22, 30, ...

3. For this Input/Output machine:



Input	Output

- a) Choose 6 numbers to input.
- b) Find each output number.
- c) Copy and complete the table.

4. For each table:

- Write the pattern rules for the input numbers and the output numbers.
- The patterns continue.
Write the next 5 input and output numbers.
- What do you do to each input number to get the output number?



a)

Input	Output
240	20
216	18
192	16
168	14
144	12

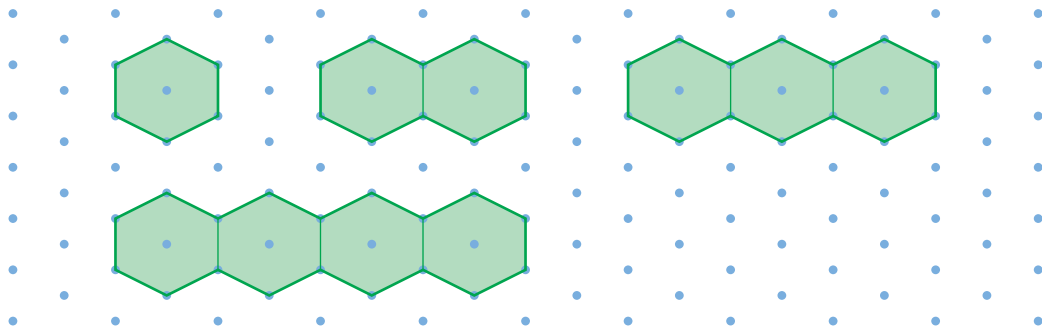
b)

Input	Output
21	189
31	279
41	369
51	459
61	549

LESSON

3

5. Congruent hexagons are combined to make new figures. Each hexagon touches no more than 2 other hexagons.



The side length of each hexagon is 1 unit.
The perimeter of each figure is recorded in a table.
The pattern continues.

Use Pattern Blocks when they help.

- a) Copy and complete the table.
- b) Write the pattern rule for the perimeters.
- c) Use the pattern to predict the perimeter of the figure with 8 hexagons.
- d) What is the perimeter of the figure with 15 hexagons?
- e) Will a figure have a perimeter of 30 units? 40 units? Explain.

Number of Hexagons	Perimeter (units)
1	6
2	10
3	
4	
5	

4

6. A magazine costs \$3.50.
- a) What is the cost of 2 magazines? 3 magazines? 4 magazines? 5 magazines? 6 magazines? Show your answers in a table.
 - b) How much would 98 magazines cost?
 - c) How many magazines can you buy with \$100?
 - d) Write your own problem about these magazines. Solve the problem.

UNIT

1

Learning Goals

- find a pattern rule for a number pattern
- identify, extend, and create patterns
- describe and model patterns
- use patterns to pose and solve problems
- use patterns in a table to make predictions

Unit Problem

Charity Fundraising



Plan an event to raise money for charity.

Include:

- A description of the event
- How much you estimate the costs will be
- How much money you expect to raise
- Tables to show any patterns in the money you expect to raise
- A poster to promote your fundraising event

Check List

Your work should show

- a detailed plan of the event
- how you calculate the amount you expect to raise
- any tables and patterns you used
- correct math language



Reflect on the Unit

Write about some of the different patterns in the unit, and how you used them to solve problems.